Transmission between International Markets & Causality

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Discussion of

Testing for Extreme Volatility Transmission with Realized Volatility Measures

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http://freakonometrics.hypotheses.org

Granger Causality

Causality 2$\rightarrow$ 1,

$$\mathbb{P}[X_{t+1}^{(1)} \in \mathcal{A} \mid \mathcal{F}_t^{(1,2)}] \neq \mathbb{P}[X_{t+1}^{(1)} \in \mathcal{A} \mid \mathcal{F}_t^{(1)}]$$

see Granger (1969), or more conveniently

$$f(x_{t+1}^{(1)}|x_t^{(1)}) \neq f(x_{t+1}^{(1)}|x_t^{(1)}, x_t^{(2)}).$$

Classical interpretation on the first moment

$$\mathbb{E}[X_{t+1}^{(1)} \mid \mathcal{F}_t^{(1,2)}] \neq \mathbb{E}[X_{t+1}^{(1)} \mid \mathcal{F}_t^{(1)}]$$

with a linear model (VAR) it is a nullity test, i.e.

$$X_{t+1}^{(1)} = \phi_{11}X_t^{(1)} + \underbrace{\phi_{12}}_{\overset{?}{=}0} X_t^{(2)} + \varepsilon_{t+1}$$

See also Taamouti *et al.* (2012) or Bouezmarni *et al.* (2009).

Volatility transmission between international markets

Volatility transmission with High Frequency Data

Use of realized volatility to estimate the true latent process of volatility

But as mentioned in Corradi, Distaso & Fernandes (2012) it is necessary to control for the effect of jumps and microstructure noise in the log-price process when testing for spillover effects in the integrated variance.

Here volatility transmission test focuses only on extreme or large values of the volatility process.

The Model

Underlying continous time model

$$d\boldsymbol{p}(t) = \boldsymbol{\mu}(t)dt + \boldsymbol{\Sigma}(t)d\boldsymbol{W}_t + \boldsymbol{\xi}d\boldsymbol{N}_t$$

with $\boldsymbol{J}(t) = \boldsymbol{\xi}(t)d\boldsymbol{N}_t$.

From an econometrician perspective, we have discrete time observed return

T = number of days and $M = \Delta^{-1}$ = number of intraday returns,

$$\boldsymbol{r}_{t+j\Delta} = \boldsymbol{p}^\star_{t+j\Delta} - \boldsymbol{p}^\star_{t+(j-1)\Delta} \text{ where } \boldsymbol{p}^\star_{t+j\Delta} = \boldsymbol{p}^\star_{t+j\Delta} + \boldsymbol{\epsilon}_{t+j\Delta}$$

Realized variance (see Barndorff-Nielsen & Shephard (2001)) is

$$\boldsymbol{RV}_{t,M} = \sum_{j=1}^{M} \boldsymbol{r}_{t-1+j\Delta}\boldsymbol{r}^\mathsf{T}_{t-1+j\Delta}$$

Inference

Quadratic variation is $\boldsymbol{QV}_{t+1} = \underbrace{\int_t^{t+1} \boldsymbol{\Sigma}(\tau)d\tau}_{\boldsymbol{IV}_{t+1}} + \underbrace{\sum_{\tau=t}^{t+1} \boldsymbol{J}(\tau)\boldsymbol{J}(\tau)^\mathsf{T} d\tau}_{\boldsymbol{JV}_{t+1}}$ A consistent nonparametric estimator of $\boldsymbol{QV}_{t+1}$ (see Barndorff-Nielsen & Shephard (2002)) is $\boldsymbol{RV}_{t,M}$, since $\boldsymbol{RV}_{t,M} \xrightarrow{\mathbb{P}} \boldsymbol{Q}_t$ as $M \to \infty$.

A consistent nonparametric estimator of $\boldsymbol{IV}_{t+1}$ (see Barndorff-Nielsen & Shephard (2002)) is $\boldsymbol{BV}_{t,M}$ (bipolar variation), with

$$\boldsymbol{BV}_{t,M} = \frac{\pi}{2} \sum_{j=2}^{M} |\boldsymbol{r}_{t-1+j\Delta}||\boldsymbol{r}^\mathsf{T}_{t-1+(j-1)\Delta}|$$

$\boldsymbol{IV}_t$ = integrated variance, with cdf $F_t(\cdot)$, and maginal quantile functions $q_{i,t}(\cdot)$. Extrem periods are defined as $z_{i,t}(\alpha) = \mathbf{1}(IV_{i,t} > q_{i,t}(\alpha))$.

For non causality $2 \to 1$, test here $H_0 : \mathbb{E}[z_{1,t}(\alpha)|\mathcal{F}_{t-1}^{(1,2)}] = \mathbb{E}[z_{1,t}(\alpha)|\mathcal{F}_{t-1}^{(1)}]$.

A model for $q_t(\alpha)$

Consider a heterogeneous autoregressive quantile model (HARQ), see Zikes & Barunik (2014)

$$\boldsymbol{q}_t(\alpha) = \boldsymbol{\theta}_1 + \boldsymbol{\theta}_2 \boldsymbol{IV}_{t-1} + \boldsymbol{\theta}_3 \overline{\boldsymbol{V}}_t^{(5)} + \boldsymbol{\theta}_4 \overline{\boldsymbol{V}}_t^{(22)} + \varepsilon_t$$

with $\overline{\boldsymbol{V}}_t^{(J)} = \frac{1}{J}\sum_{j=1}^{J} \boldsymbol{IV}_{t-j}$.

Using Koenker & Bassett Jr (1978), estimate $\boldsymbol{\theta}_i$ by solving

$$\underset{\boldsymbol{\theta}\in\mathbb{R}^4}{\text{argmin}} \left\{ \frac{1}{T}\sum_{t=2}^{T} (\alpha - \mathbf{1}(u_{i,t} < 0)) \cdot u_{i,t} \right\} \text{ where } u_{i,t} = \widehat{IV}_{i,t} - q_{i,t}(\alpha, \boldsymbol{\theta})$$

and then set $\widehat{z}_{i,t}(\alpha) = \mathbf{1}(\widehat{IV}_{i,t} > \widehat{q}_{i,t}(\alpha))$.

Cross-covariance at lag-order h

Then compute cross-covariances

$$\widehat{\rho}_h^{(1,2)}=\frac{\widehat{C}_h^{(1,2)}}{\alpha(1-\alpha)} \text{ with } \widehat{C}_h^{(1,2)}=\frac{1}{T-j}\sum_{t=1+j}^{T}[\widehat{z}_{1,t}(\alpha)-(1-\alpha)][\widehat{z}_{2,t}(\alpha)-(1-\alpha)]$$

and consider Ljung & Box (1978)'s statistics,

$$\widehat{Q}_H^{(1,2)}=T(T+2)\sum_{h=1}^{H}\frac{[\widehat{\rho}_h^{(1,2)}]^2}{T-h}$$

Under H_0, $\widehat{Q}_H^{(1,2)}\sim\xi^2(H)$ as $T,M,\to\infty$.

From asymptotics to finite sample

$M = 23,400$ (1 second sampling frequency and a trading day of 6.5 hours)

Simulations based on log-volatility follows a stationary three-regime smooth-transition heterogeneous autoregressive (HARST) model